Entanglement transfer from electron spins to photons in spin light-emitting diodes containing quantum dots

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We show that electron recombination using positively charged excitons in single quantum dots provides an efficient method to transfer entanglement from electron spins onto photon polarizations. We propose a scheme for the production of entangled four-photon states of GHZ type. From the GHZ state, two fully entangled photons can be obtained by a measurement of two photons in the linear polarization basis, even for quantum dots with observable fine structure splitting for neutral excitons and significant exciton spin decoherence. Because of the interplay of quantum mechanical selection rules and interference, maximally entangled electron pairs are converted into maximally entangled photon pairs with unity fidelity for a continuous set of observation directions. We describe the dynamics of the conversion process using a master-equation approach and show that the implementation of our scheme is feasible with current experimental techniques.

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I. INTRODUCTION

Spin light-emitting diodes (spin-LEDs),[1,2,3,4,5,6,7,8] in which electron recombination is accompanied by the emission of a photon with well-defined circular polarization, provide an efficient interface between electron spins and photons. The operation of such devices at the single-photon level would allow one to convert the quantum state of an electron encoded in its spin state into that of a photon with a wide range of possible applications. In view of quantum information schemes, converting spin into photon quantum states corresponds to a conversion of localized into flying qubits, which can be transmitted over long distances and could overcome limitations caused by the short-range nature of the electron exchange interaction.[7] On a more fundamental level, the photon polarization can be readily measured experimentally such that an interface between spins and photons will allow one to measure quantum properties of the spin system via the photons generated on recombination. More specifically, entanglement of electron spins could be demonstrated not only in current noise[9,10] but also via photon polarizations which allows one to test Bell's inequalities.[11]

In this work, we show that nonlocal spin-entangled electron pairs that recombine in single quantum dots contained in spatially separated spin-LEDs are converted into polarization-entangled photon states. In addition to its applications in quantum communication, this transfer can be used to characterize the output of an electron spin entangler[12,13,14,15,16,17,18,19] in a setup as shown in Fig. 1. Furthermore, such a setup acts as a deterministic source of polarization-entangled photon pairs. Recently, the decay of biexcitons in single quantum dots has been proposed for the production of entangled photons.[20,21] However, several experiments[22,23,24,25,26] have only shown polarization correlation but not entanglement of the photons. The fine structure splitting $\delta_{\rm ehx}$ of the bright exciton ground state[27] has been identified to be crucial for the lack of entanglement: Firstly, the polarization-entangled photons are also entangled in energy if $\delta_{\rm ehx}$ is larger than the exciton linewidth.[28] Secondly, for $\delta_{\rm ehx} \neq 0$ the exciton spin relaxation rate due to phonons $1/T_{1,X}$ is enhanced[29] and leads to an increased decoherence rate $1/T_{2,X} = 1/2T_{1,X} + 1/T_{\varphi,X}$, where $1/T_{\varphi,X}$ is the pure decoherence rate. To overcome these difficulties we propose to use positively charged excitons (X^+), for which $\delta_{\rm ehx} = 0$ up to small corrections. Moreover, we demonstrate that the antisymmetric hole ground state of the X^+ enables the production of entangled four-photon states. We study the transfer of entanglement for different photon emission directions by calculating the von Neumann entropy. Due to quantum mechanical interference, the fidelity of this process approaches unity not only for photon emission along the spin quantization axis, but for a continuous set of observation directions. The relaxation and decoherence of the electron spins in the leads is modeled using a master equation and it is quantified by the fidelity of the entangled state.

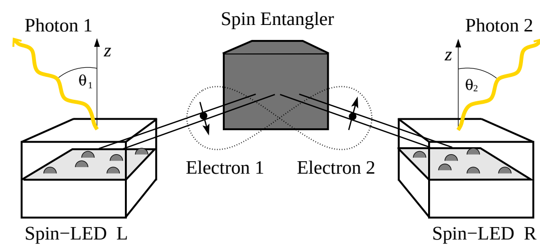

FIG. 1: (Color online) Schematic setup for the transfer of entanglement between electrons and photons. An electron entangler (gray box) injects a pair of spin-entangled electrons into two current leads. The electrons recombine individually in one quantum dot located in the left (L) and one in the right (R) spin-LED and give rise to the emission of two photons.

This work is organized as follows. In Sec. II we describe the dynamics of the conversion process. In Sec. III we focus on the microscopic expressions for the involved optical transitions, leading to entangled four-photon and two-photon states. In Sec. IV we quantify the entanglement of the two-photon state as a function of the emission angles. We conclude in Sec. V.

II. DYNAMICS OF THE CONVERSION PROCESS

The effective Hamiltonian of the system is given by

$$H = H_L + H_R + H_{\mathrm{rad}} + H_{\mathrm{int}}, \quad (1)$$

where $H_\alpha = \mathbf{p}^2/2m + V_{\mathrm{qd}}(\mathbf{r})$ is the Hamiltonian of the quantum dot $\alpha = L, R$ with confinement potential $V_{\mathrm{qd}}(\mathbf{r})$. The Hamiltonian of the radiation field is $H_{\mathrm{rad}} = \sum_{\mathbf{k},\lambda} \hbar\omega_k a^\dagger_{\mathbf{k}\lambda} a_{\mathbf{k}\lambda}$ and $H_{\mathrm{int}} = -e\mathbf{A}\cdot\mathbf{p}/m_0 c = H_{\mathrm{em}} + H.c.$ is the optical interaction term, which is linear in both the vector potential $\mathbf{A}$ and the electron momentum $\mathbf{p}$ and can be decomposed into a photon emission term H_{em} and its Hermitian conjugate. For simplicity, we assume that the dots L and R are identical, with cubic crystal structure and with aligned main crystal axes. We choose the z axis parallel to the quantum dot growth direction (e.g., [001]). If the quantum dot confinement is stronger in the z direction than in the xy plane, z defines the spin quantization axis and heavy-hole (hh) and light-hole (lh) states are energetically split by $\Delta_{\mathrm{hh-lh}}$ (typically $\Delta_{\mathrm{hh-lh}} \sim 10\,\mathrm{meV}$). We consider a hh ground state, with angular momentum projection $\pm 3/2$ in terms of electron quantum numbers. We further focus on the strong-confinement regime, where the dot radius is smaller than the exciton Bohr radius.

The quantum dots in both spin-LEDs are prepared in a state $|\chi_\alpha\rangle$, where two excess holes occupy the lowest hh level in each dot. This initial state, which can be generated by applying an appropriate bias voltage across the LED, has several advantages. Firstly, electrons with arbitrary spin states can recombine optically, as demonstrated for electron spin detection in a recent experiment.[5] Secondly, the z component of the total hole spin vanishes. This is a consequence of the fact that in quantum dots the hh-lh exciton mixing due to the electron-hole exchange interaction Δ_{ehx} is determined by a small parameter $\Delta_{\mathrm{ehx}}/\Delta_{\mathrm{hh-lh}} \sim 0.01$. Thus, injected spin-polarized electrons give rise to circularly polarized X^+ luminescence. This remains true for dots with asymmetric confinement in the xy plane, in stark contrast to the case with an electron and only one hole in the dot,[27] where the good exciton eigenstates are horizontally polarized and are split in energy typically by $\delta_{\mathrm{ehx}} \sim 0.1\,\mathrm{meV}$. Thus, the electron-hole exchange interaction can be neutralized by initially providing *two* holes. Interband mixing (e.g., hh and lh states) in strongly anisotropic dots reduces the maximum circular polarization of photons emitted from spin-polarized electrons[4] and reduces the fidelity of our scheme. However, because the interband transition probability for lh states is three times smaller than that for hh states, and hh-lh mixing is typically controlled by some small parameter in slightly elliptical dots,[27] we neglect lh transitions.

A. Electron injection and photon emission

We first describe the dynamics of the electron injection and recombination in the two dots using a master equation. The rate for the injection and the subsequent relaxation of electrons into the conduction band ground state in the dot α is denoted by $W_{e\alpha}$. It has been demonstrated that this entire process is spin conserving and occurs much faster than the optical recombination[5,6], which is described by the rates $W_{p\alpha}$. Typically, $W_{p\alpha} \sim 1\,(\mathrm{ns})^{-1}$ and $W_{e\alpha} \sim 0.1\,(\mathrm{ps})^{-1}$ for the incoherent transition rates. We solve the master equation for the classical occupation probabilities and obtain the probability that two photons are emitted after the injection of two electrons into the dots at $t = 0$,

$$P_{2p} = \prod_{\alpha=L,R} \frac{W_{e\alpha}(1-e^{-tW_{p\alpha}}) - W_{p\alpha}(1-e^{-tW_{e\alpha}})}{W_{e\alpha} - W_{p\alpha}}. \quad (2)$$

For $W_{p\alpha} \ll W_{e\alpha}$, $P_{2p} \approx \prod_{\alpha=L,R}(1-e^{-tW_{p\alpha}})$. After photon emission, bipartite photon entanglement is achieved by a measurement of the hole spins as we describe below and the initial state is finally restored by injection of two holes into each of the two dots. We estimate the production rate of entangled photons in a setup to test some of the proposed electron entanglers.[12,13,14,15,16,17,18,19] For example, electron spin singlets $|\Psi^-\rangle = (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$ are produced by the Andreev entangler[12] with an average time separation $\Delta t \sim 10^{-5}$s, while for the entangler based on three quantum dots,[17] $\Delta t \sim 10^{-8}$s. The two electrons of a singlet typically are injected into the current leads with a relative time delay $\tau \simeq 10^{-13}$s for both of these entanglers. Because $\tau, W_{p\alpha}^{-1} \ll \Delta t$, photons originating from a single pair of entangled electrons can be identified with high reliability. In the steady state, the generation rate of entangled photons is determined by the rate at which entangled electron pairs leave the entangler, $1/\Delta t$.

B. Electron spin dynamics

Relaxation and decoherence is taken into account for the two spins by the single-spin Bloch equation.[30] Given that the electrons are in different leads, they interact with different environments (during times t and t', respectively). Therefore, we consider different magnetic fields $\mathbf{h}$ and $\mathbf{h}'$, enclosing an angle β, each acting on an individual spin. We calculate the two-spin density matrix $\chi(t,t')$ and obtain for the singlet fidelity $f = 4\langle\Psi^-|\chi(t,t')|\Psi^-\rangle$

(given in Ref.[30] for $t = t'$ and $\beta = 0$),

$$\begin{aligned} f &= 1 - \cos\beta\, aa'PP' + e_1\left[e_2'\sin^2\beta\cos(h't') + e_1'\cos^2\beta\right] \\ &\quad + e_2e_1'\sin^2\beta\cos(ht) + e_2e_2'\left[2\cos\beta\sin(ht)\sin(h't')\right. \\ &\quad \left. + \left(\cos^2\beta + 1\right)\cos(ht)\cos(h't')\right], \end{aligned} \tag{3}$$

where for the first (second) spin $e_i = e^{-t/T_i}$ ($e_i' = e^{-t'/T_i'}$), $a = 1 - e_1$ ($a' = 1 - e_1'$), P (P') is the equilibrium polarization, and T_2 and T_1 (T_2' and T_1') are the spin decoherence and relaxation times, respectively. For $t \ll T_1, T_2$ and $t' \ll T_1', T_2'$ (in bulk GaAs $T_2 \sim 100\,\mathrm{ns}$ has been measured[31] and, typically, $T_1 \gg T_2$), the electrons form a nonlocal spin-entangled state after their injection into the dots L and R and after their subsequent relaxation to the single-electron orbital ground states $\phi_{c\alpha}(\mathbf{r}_{c\alpha}, \sigma)$. A local rotation of one of the two spins in the leads (for $\mathbf{h} \neq \mathbf{h}'$) enables a transformation of $|\Psi^-\rangle$ into another (maximally entangled) Bell state $|\Psi^+\rangle = (|\uparrow\downarrow\rangle + |\downarrow\uparrow\rangle)/\sqrt{2}$ or $|\Phi^\pm\rangle = (|\uparrow\uparrow\rangle \pm |\downarrow\downarrow\rangle)/\sqrt{2}$. This can be achieved, e.g., by controlling the local Rashba spin-orbit interaction in the current leads.[10,30]

III. OPTICAL TRANSITIONS

The optical recombination processes of the two electrons occur independently, except for the entanglement of the spin wave functions. We consider one single branch $\alpha = L, R$ of the apparatus and omit the index α. The state of the single quantum dot which is charged with two hhs in the orbital ground state and into which a single electron with spin σ has been injected is given by

$$|e, \sigma\rangle = \int \mathrm{d}^3 r_c \phi_c^*(\mathbf{r}_c, \sigma) b_{c\sigma}^\dagger(\mathbf{r}_c)|\chi\rangle. \tag{4}$$

Here, $b_{c\sigma}^\dagger(\mathbf{r}_c)$ creates an electron with spin $S_z = \sigma/2 = \pm 1/2$ at $\mathbf{r}_c$ in the ground state of the dot, $|\chi\rangle = \sum_{\tau\neq\tau'} \int \mathrm{d}^3 r_{v1} \mathrm{d}^3 r_{v2} \phi_v(\mathbf{r}_{v1}, \tau; \mathbf{r}_{v2}, \tau') b_{v\tau}(\mathbf{r}_{v1}) b_{v\tau'}(\mathbf{r}_{v2})|g\rangle$, where $|g\rangle$ is the electrostatically neutral ground state of the quantum dot, and $\phi_v(\mathbf{r}_{v1}, \tau; \mathbf{r}_{v2}, \tau')$ is the orbital part of the two-hole wave function. In the strong-confinement regime where Coulomb correlations are negligible, ϕ_v is a product of the single-particle valence band states. The labels τ, τ' denote the hh spin component $S_z = \tau/2 = \pm 1/2$ that factor out for angular momentum $J_z = \pm 3/2$. We now calculate the emission matrix element $\langle f|H_{\mathrm{em}}|i\rangle$ with initial state $|i\rangle = |e, \sigma\rangle \otimes |\ldots, n_{\mathbf{k}\lambda}, \ldots\rangle$ and final state $|f\rangle = b_{v\tau'}(\mathbf{r}_{v2})|g\rangle \otimes |\ldots, n_{\mathbf{k}\lambda} + 1, \ldots\rangle$, where $|\ldots, n_{\mathbf{k}\lambda}, \ldots\rangle$ is a Fock state of the electromagnetic field, typically the photon vacuum. Because of quantum mechanical selection rules, the optical transitions connect only states with the same spin such that $\tau' \neq \sigma$. In the envelope-function and dipole approximations,[32]

$$|\langle f|H_{\mathrm{em}}|i\rangle| = \frac{e}{m_0 c} A_0(\omega_k)\sqrt{n_{\mathbf{k}\lambda} + 1}\, |\mathbf{e}_{\mathbf{k}\lambda}^* \cdot \mathbf{p}_{cv}^* C_{eh}|, \tag{5}$$

where $\mathbf{p}_{cv}^* = \mathbf{p}_{vc}$ is the inter-band momentum matrix element, $\mathbf{e}_{\mathbf{k}\lambda}$ is the unit polarization vector with $\lambda = \pm 1$ for circular polarization $|\sigma_\pm\rangle$, $A_0(\omega_k) = (\hbar/2\epsilon\epsilon_0\omega_k V)^{1/2}$, and $C_{eh} = \int \mathrm{d}^3 r\, \psi_c^*(\mathbf{r}, \sigma)\psi_v(\mathbf{r}, \sigma)$, where ψ_n is the envelope function of a carrier in the band $n = c, v$. For cubic symmetry, $\mathbf{e}_{\mathbf{k}\lambda}^* \cdot \mathbf{p}_{cv}^* = p_{cv}(\cos\theta - \sigma\lambda)e^{-i\sigma\phi}/2 \equiv p_{cv} m_{\sigma\lambda}(\theta, \phi)$, where θ and ϕ are the polar and the azimuthal angle of the photon emission direction, respectively. With the transition $|e, \sigma\rangle \to b_{v-\sigma}(\mathbf{r}_{v2})|g\rangle$, a photon

$$|\sigma, \theta, \phi\rangle = N(\theta)(m_{\sigma,+1}(\theta, \phi)|\sigma_+\rangle + m_{\sigma,-1}(\theta, \phi)|\sigma_-\rangle) \tag{6}$$

is emitted into the direction (θ, ϕ). Here, $N(\theta) = [2/(1 + \cos^2\theta)]^{1/2}$ is a normalization factor. Eq. (6) shows that for $\theta = 0$, a spin-up ($\sigma = +1$) electron generates a $|\sigma_-\rangle$ photon, whereas a $|\sigma_+\rangle$ photon is obtained from a spin-down ($\sigma = -1$) electron. The admixture of the opposite circular polarization increases with θ, leading to linear polarization for $\theta = \pi/2$. For $\theta \neq 0$, the spin-inverted states $|+1, \theta, \phi\rangle$ and $|-1, \theta, \phi\rangle$ have interchanged coefficients for $|\sigma_+\rangle$ and $|\sigma_-\rangle$, up to a relative phase determined by the (global) phase factors $\exp(-i\sigma\phi)$. Note that in two-photon states the azimuthal angles thus can provide a *relative* phase, as we exploit below.

A. Entangled four-photon state

The two photons produced at recombination are entangled with the two holes which remain in the dots, due to the antisymmetric hole ground state. By injecting a pair of electrons with spins polarized in the xy plane into the dots,[33] a four-photon state of the Greenberger-Horne-Zeilinger (GHZ) type[34] can be produced if $T_{1,X}$ and $T_{2,X}$ exceed the exciton lifetime τ_X. For the two polarized electrons, only the electron spin orientation in z direction which satisfies the optical selection rules contributes to the optical transition, respectively. For circularly polarized photons emitted along z, the electron Bell states give rise to the photon states

$$|\Psi^\pm\rangle \to |\sigma_+\sigma_-\sigma_-\sigma_+\rangle \pm |\sigma_-\sigma_+\sigma_+\sigma_-\rangle, \tag{7}$$

$$|\Phi^\pm\rangle \to |\sigma_-\sigma_-\sigma_+\sigma_+\rangle \pm |\sigma_+\sigma_+\sigma_-\sigma_-\rangle, \tag{8}$$

where the first two entries indicate the first photon pair (L,R) and the third and fourth entry the second photon pair (L,R), respectively. Normalization has been omitted for simplicity. Yet, the second photon pair is generated by neutral excitons and is thus exposed to the same problems as the biexciton decay cascade in asymmetric quantum dots. Here, a cavity can be used to maintain the GHZ state since the energy entanglement of the second photon pair can be erased,[28] and τ_X can be shortened due to the Purcell effect to reduce exciton polarization decoherence.

B. Entangled two-photon state

Full *bipartite* photon entanglement of the first photon pair is obtained, e.g., by directing the second photon pair via secondary optical paths to a linear polarization measurement which is performed *before* the first photon pair is measured,[35] see Fig. 2 (a). Even different bases $\{|H\rangle, |V\rangle\}$ and $\{|H'\rangle, |V'\rangle\}$ can be chosen for the two photons of the second pair. Note that the electron-hole exchange interaction in elliptical dots assists this projection into linearly polarized eigenstates (along the major and the minor axis of the dots, respectively) already during the lifetime of the remaining two excitons. While the loss of (linear) polarization coherence is tolerable for these excitons, $T_{1,X} > \tau_X$ is required for entanglement of the first photon pair. This suggests that the scheme presented here can be realized with typical quantum dots, see Ref.[29] and references therein.

If the second photon pair is measured in the state $|HH'\rangle$ or $|VV'\rangle$, the electron Bell states have given rise to the two-photon states

$$\begin{aligned}|\Psi^{\pm}\rangle \;\rightarrow\; & |+1,\theta_1,\phi_1\rangle_L\,|-1,\theta_2,\phi_2\rangle_R \\ & \pm\,|-1,\theta_1,\phi_1\rangle_L\,|+1,\theta_2,\phi_2\rangle_R, \qquad (9)\\ |\Phi^{\pm}\rangle \;\rightarrow\; & |+1,\theta_1,\phi_1\rangle_L\,|+1,\theta_2,\phi_2\rangle_R \\ & \pm\,|-1,\theta_1,\phi_1\rangle_L\,|-1,\theta_2,\phi_2\rangle_R. \qquad (10)\end{aligned}$$

Here, normalization has been omitted for simplicity. If the second photon pair is measured as $|HV'\rangle$ or $|VH'\rangle$, $\pm$ is replaced by $\mp$ on the right-hand side of Eqs. (9) and (10).

Obviously, above two-photon states (9) and (10) are maximally entangled for $\theta_1 = \theta_2 = 0$. For $\theta_1 = \theta_2 \in (0, \pi/2)$, the total relative phase factor between the two-photon states in Eq. (9) is $\exp(i\gamma + 2i\Delta\phi)$. Here, $\Delta\phi = \phi_1 - \phi_2$, and the relative phase of the two-electron states is $\gamma = \pi$ for $|\Psi^-\rangle$ and $\gamma = 0$ for $|\Psi^+\rangle$. For Eq. (10), the relative phase factor is $\exp[i\gamma + 2i(\phi_1 + \phi_2)]$, with $\gamma = \pi$ for $|\Phi^-\rangle$ and $\gamma = 0$ for $|\Phi^+\rangle$. By tuning the relative phase factors in Eqs. (9) and (10) to -1, two circularly polarized photons can be recovered for $\theta_1 = \theta_2 \in (0, \pi/2)$ from the elliptically polarized single-photon states due to quantum mechanical interference.[36] Thus, maximal entanglement is transferred from two electron spins to the polarizations of two photons for certain ideal emission angles. For $|\Psi^-\rangle$ ($|\Psi^+\rangle$), $\Delta\phi = 0$ ($\Delta\phi = \pi/2$) needs to be satisfied modπ, whereas the condition for $|\Phi^-\rangle$ ($|\Phi^+\rangle$) is $\phi_1 + \phi_2 = 0$ ($\phi_1 + \phi_2 = \pi/2$) modπ. For $\theta_1 = \theta_2 = \pi/2$ these two-photon states vanish completely due to destructive interference.

IV. PHOTON ENTANGLEMENT AS A FUNCTION OF EMISSION DIRECTIONS

For arbitrary emission directions of the two photons, the degree of polarization entanglement can be quantified by the von Neumann entropy $E = -\mathrm{tr}_2(\tilde{\rho}\log_2\tilde{\rho})$.

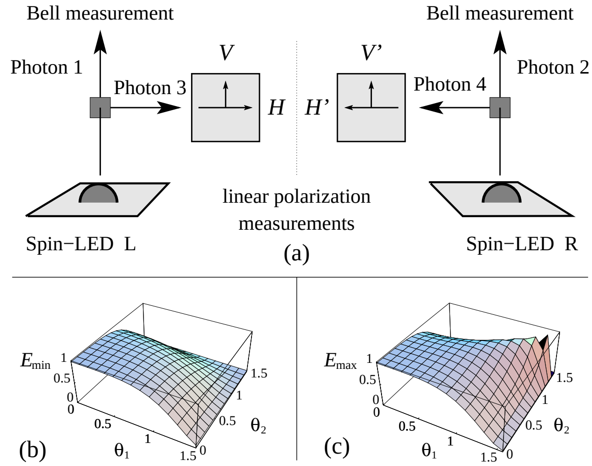

FIG. 2: (Color online) (a) Schematic setup to obtain bipartite entanglement of photons 1 and 2 by measuring the photons 3 and 4 of the GHZ state in bases of linear polarizations H, V and H', V', respectively (see the text). In (b) and (c), we show the von Neumann entropy (b) $E = E_{\min}$ and (c) $E = E_{\max}$ as a function of the polar angles θ_1 and θ_2 for photon emission. E oscillates between (b) and (c) as a function of ϕ_1 and ϕ_2, as explained in the text. The photon-polarization entanglement is maximal for $\theta_1 = \theta_2 = 0$, whereas for $\theta_i = \pi/2$ entanglement is absent. In (c), $E_{\max} = 1$ for the continuous set of directions $\theta_1 = \theta_2 \in [0, \pi/2)$.

Here, $\tilde{\rho} = \mathrm{tr}_1\rho$ is the reduced density matrix of the two-photon state ρ with the trace tr_1 taken over photon 1. For a maximally entangled two-photon state $E = 1$, while $E = 0$ represents a pure state $\tilde{\rho}$ (which implies the absence of bipartite entanglement). If the two electrons recombine after times much shorter than the spin lifetimes T_1, T_1', T_2, T_2', E oscillates for Eq. (9) as a function of $\Delta\phi$ of the two emitted photons between a minimal value,

$$E_{\min} = \log_2(1 + x_1x_2) - \frac{x_1x_2\log_2(x_1x_2)}{1 + x_1x_2}, \quad (11)$$

and a maximal value,

$$E_{\max} = \log_2(x_1 + x_2) - \frac{x_1\log_2(x_1)}{x_1 + x_2} - \frac{x_2\log_2(x_2)}{x_1 + x_2}, \quad (12)$$

where $x_i = \cos^2\theta_i$, which is (only) obtained for the ideal angles ϕ_1 and ϕ_2 mentioned above; see Fig. 2 (b) and (c). For Eq. (10), E oscillates between $E_{\min}$ and $E_{\max}$ as a function of $\phi_1 + \phi_2$. As expected, $E_{\max} = 1$ for all $\theta_1 = \theta_2 \in [0, \pi/2)$. The discontinuity in $E_{\max}$ for $\theta_1 = \theta_2 = \pi/2$ is due to the vanishing two-photon state.

V. CONCLUSIONS

We have studied the transfer of entanglement from electron spins to photon polarizations. We have discussed the generation of entangled four-photon and two-photon states via the injection of spin-entangled electrons

into quantum dots charged with two excess holes. We have proposed a scheme to achieve complete entanglement transfer from two electron spins to two photons. We have shown that this scheme can even be realized with quantum dots exhibiting an exciton exchange splitting. We have shown the dependence of the photon entanglement on the emission angles and identified the conditions for maximal entanglement. This offers the possibility to efficiently test Bell's inequalities for electron spins. In addition, our results show that a continuous set of directions exist along which entanglement is maximal. Finally, similar schemes to produce entangled photons can be realized using two tunnel-coupled dots[37] instead of two isolated dots. In such a setup, it is essential that tunnel coupling is provided for the conduction-band electrons, whereas the valence-band holes are not tunnel coupled and thus localized in the individual dots. After a positively charged exciton is created in each of the two dots, the spin entanglement is provided from the singlet ground state of the delocalized electrons and can be transferred to the photons, similarly as described in this work.

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[1] R. Fiederling, M. Keim, G. Reuscher, W. Ossau, G. Schmidt, A. Waag, and L. W. Molenkamp, Nature **402**, 787 (1999).
[2] Y. Ohno, D. K. Young, B. Beschoten, F. Matsukura, H. Ohno, and D. D. Awschalom, Nature **402**, 790 (1999).
[3] Y. Chye, M. E. White, E. Johnston-Halperin, B. D. Gerardot, D. D. Awschalom, and P. M. Petroff, Phys. Rev. B **66**, 201301(R) (2002).
[4] C. E. Pryor and M. E. Flatté, Phys. Rev. Lett. **91**, 257901 (2003).
[5] K. Gündoḡdu, K. C. Hall, T. F. Boggess, D. G. Deppe, and O. B. Shchekin, Appl. Phys. Lett. **84**, 2793 (2004).
[6] J. Seufert, G. Bacher, H. Schömig, A. Forchel, L. Hansen, G. Schmidt, and L. W. Molenkamp, Phys. Rev. B **69**, 035311 (2004).
[7] *Semiconductor Spintronics and Quantum Computation*, edited by D. D. Awschalom, D. Loss, and N. Samarth (Springer-Verlag, Berlin, 2002).
[8] M. Kroutvar, Y. Ducommun, D. Heiss, M. Bichler, D. Schuh, G. Abstreiter, and J. J. Finley, Nature **432**, 81 (2004).
[9] G. Burkard, D. Loss, and E. V. Sukhorukov, Phys. Rev. B **61**, R16 303 (2000).
[10] J. C. Egues, G. Burkard, and D. Loss, Phys. Rev. Lett. **89**, 176401 (2002).
[11] J. Bell, Physics **1**, 195 (1965).
[12] P. Recher, E. V. Sukhorukov, and D. Loss, Phys. Rev. B **63**, 165314 (2001).
[13] G. B. Lesovik, T. Martin, and G. Blatter, Eur. Phys. J. B, **24**, 287 (2001).
[14] P. Recher and D. Loss, Phys. Rev. B **65**, 165327 (2002).
[15] C. Bena, S. Vishveshwara, L. Balents, and M. P. A. Fisher, Phys. Rev. Lett. **89**, 037901 (2002).
[16] V. Bouchiat, N. Chtchelkatchev, D. Feinberg, G. B. Lesovik, T. Martin, and J. Torrés, Nanotechnology **14**, 77 (2003).
[17] D. S. Saraga and D. Loss, Phys. Rev. Lett. **90**, 166803 (2003).
[18] P. Recher and D. Loss, Phys. Rev. Lett. **91**, 267003 (2003).
[19] D. S. Saraga, B. L. Altshuler, D. Loss, and R. M. Westervelt, Phys. Rev. Lett. **92**, 246803 (2004).
[20] O. Benson, C. Santori, M. Pelton, and Y. Yamamoto, Phys. Rev. Lett. **84**, 2513 (2000).
[21] E. Moreau, I. Robert, L. Manin, V. Thierry-Mieg, J. M. Gérard, and I. Abram, Phys. Rev. Lett. **87**, 183601 (2001).
[22] A. Kiraz, S. Fälth, C. Becher, B. Gayral, W. V. Schoenfeld, P. M. Petroff, L. Zhang, E. Hu, and A. Imamoḡlu, Phys. Rev. B **65**, 161303(R) (2002).
[23] C. Santori, D. Fattal, M. Pelton, G. S. Solomon, and Y. Yamamoto Phys. Rev. B **66**, 045308 (2002).
[24] R. M. Stevenson, R. M. Thompson, A. J. Shields, I. Farrer, B. E. Kardynal, D. A. Ritchie, and M. Pepper, Phys. Rev. B **66**, 081302(R) (2002).
[25] V. Zwiller, P. Jonsson, H. Blom, S. Jeppesen, M.-E. Pistol, L. Samuelson, A. A. Katznelson, E. Yu. Kotelnikov, V. Evtikhiev, and G. Björk, Phys. Rev. A **66**, 053814 (2002).
[26] S. M. Ulrich, S. Strauf, P. Michler, G. Bacher, and A. Forchel, Appl. Phys. Lett. **83**, 1848 (2003).
[27] T. Takagahara, Phys. Rev. B **62**, 16 840 (2000).
[28] T. M. Stace, G. J. Milburn, and C. H. W. Barnes, Phys. Rev. B **67**, 085317 (2003).
[29] E. Tsitsishvili, R. v. Baltz, and H. Kalt, Phys. Rev. B **67**, 205330 (2003).
[30] G. Burkard and D. Loss, Phys. Rev. Lett. **91**, 087903 (2003).
[31] J. M. Kikkawa and D. D. Awschalom, Phys. Rev. Lett. **80**, 4313 (1998).
[32] O. Gywat, G. Burkard, and D. Loss, Phys. Rev. B **65**, 205329 (2002).
[33] To switch between the production of entangled and polarized electron pairs, a double quantum dot can be used with tunable exchange splitting J and to which an in-plane magnetic field $B_\perp$ is applied[9]. For J smaller (larger) than the Zeeman energy, the two-electron ground state is a triplet with spins along $B_\perp$ (a singlet). Alternatively, for subsequent injection of *two* entangled electron pairs, $\pm \to +$ on the right-hand side of Eqs. (7)–(10).
[34] A. Peres, *Quantum Theory: Concepts and Methods*, (Kluwer, Dordrecht, 1998).
[35] An alternative suggestion by A. Imamoḡlu (private communication) is to perform a Hadamard operation on the hh states which are left in the dots after emission of the first photon pair [e.g., via an optical Raman transition, see A. Imamoḡlu, D. D. Awschalom, G. Burkard, D. P. DiVincenzo, D. Loss, M. Sherwin, and A. Small, Phys. Rev. Lett. **83**, 4204 (1999)], followed by a hole-spin measurement along z, e.g., via state-selective absorption of circularly polarized photons.
[36] Such ideal angles can analogously be found for the four-photon GHZ states.

[37] O. Gywat, Ph.D. thesis, University of Basel, February 2005.